Math 137: Algebraic Geometry

Spring 2021

Problem set #11

due Friday, April 30 at noon

On this problem set, K is an algebraically closed field.

Problem 1. For $t \in K$, consider the polynomials $f = y^2 - x(x-1)^2$ and $g_t = y - t(x+1)$ in $K[X,Y]$. Compute the points of intersection $P \in V(f, g_t)$ and the intersection numbers $I_P(f, g_t)$.

Problem 2. Let $\varphi : K^2 \to K^2$ be a bijective affine linear map, let $f, g \in K[X,Y]$ and $P \in K^2$. Show that $I_{\varphi(P)}(f,g) = I_P(f \circ \varphi, g \circ \varphi)$. (In other words, intersection numbers are invariant under affine linear transformations.)

Problem 3. Let $a, b \geqslant 1$ and $c \geqslant ab$. Furthermore, let $P \in K^2$. Show that there are polynomials $f, g \in K[X,Y]$ such that $m_P(f) = a$ and $m_P(g) = b$ and $I_P(f,g) = c$.

Problem 4. Let V be an irreducible algebraic subset of K^n and let $P \in V$. Show that there is a bijection between the set of irreducible algebraic subsets W of V containing P and the set of prime ideals J of $\mathcal{O}_{V,P}$, where a set W corresponds to the ideal of $\mathcal{O}_{V,P}$ generated by

$$I_V(W) := \{f \in \Gamma(V) \mid \forall Q \in W : f(Q) = 0\} \subseteq \Gamma(V) \subseteq \mathcal{O}_{V,P}.$$