Math 286X: Arithmetic Statistics

Spring 2020

References

References

There is no official textbook for this course, but here are some good references:

- *Problems in Analytic Number Theory* by M. Ram Murty contains a nice introduction to Dirichlet series (chapters 1–3) and sieves (chapter 9).
- Melanie Matchett Wood's notes *Asymptotics for number fields and class groups* from the Arizona Winter School 2014: `http://swc.math.arizona.edu/aws/2014/index.html`

The following are some useful references for specific topics:

Squarefree values of polynomials Degree three [Hoo67]; using the ABC conjecture [Gra98]

Ekedahl Sieve [Eke91]

Large sieve [Ser97, chapter 12]

Counting ideals [ME05, chapter 11]

Parametrization of cubic rings [Lev14], [DH71]

Geometry of numbers [Cas97], [Sie89]

Lattice-point counting [Wid10, section 5], [Sch68], [Dav51], [Dav64]

Total number of ideal classes in imaginary quadratic orders [Mer74, section 4]

Counting number fields of fixed degree [Sch95], [EV06]

Counting number fields of degree three [DH71]

Counting number fields of degree four [Bha04], [Bha05]

Counting number fields of degree five [Bha08], [Bha10]

Counting local fields [Ser78], [Bha07], [Ked07]

p**-adic integration** [Igu00]

Malle's conjecture [Mal02], [Mal04].

I will mention more references (mostly papers) as the course progresses.

References

[Bha04] Manjul Bhargava. "Higher composition laws. III. The parametrization of quartic rings". In: *Ann. of Math. (2)* 159.3 (2004), pp. 1329–1360. ISSN: 0003-486X. DOI: `10.4007/annals.2004.159.1329`. URL: `https://doi.org/10.4007/annals.2004.159.1329`.

[Bha05] Manjul Bhargava. "The density of discriminants of quartic rings and fields". In: *Ann. of Math. (2)* 162.2 (2005), pp. 1031–1063. ISSN: 0003-486X. DOI: `10.4007/annals.2005.162.1031`. URL: `https://doi.org/10.4007/annals.2005.162.1031`.

[Bha07] Manjul Bhargava. "Mass formulae for extensions of local fields, and conjectures on the density of number field discriminants". In: *Int. Math. Res. Not. IMRN* 17 (2007), Art. ID rnm052, 20. ISSN: 1073-7928. DOI: `10.1093/imrn/rnm052`. URL: `https://doi.org/10.1093/imrn/rnm052`.

[Bha08] Manjul Bhargava. "Higher composition laws. IV. The parametrization of quintic rings". In: *Ann. of Math. (2)* 167.1 (2008), pp. 53–94. ISSN: 0003-486X. DOI: `10.4007/annals.2008.167.53`. URL: `https://doi.org/10.4007/annals.2008.167.53`.

[Bha10] Manjul Bhargava. "The density of discriminants of quintic rings and fields". In: *Ann. of Math. (2)* 172.3 (2010), pp. 1559–1591. ISSN: 0003-486X. DOI: `10.4007/annals.2010.172.1559`. URL: `https://doi.org/10.4007/annals.2010.172.1559`.

[Cas97] J. W. S. Cassels. *An introduction to the geometry of numbers*. Classics in Mathematics. Corrected reprint of the 1971 edition. Springer-Verlag, Berlin, 1997, pp. viii+344. ISBN: 3-540-61788-4.

[Dav51] H. Davenport. "On a principle of Lipschitz". In: *J. London Math. Soc.* 26 (1951), pp. 179–183. ISSN: 0024-6107. DOI: 10.1112/jlms/s1-26.3.179. URL: https://doi-org.ezp-prod1.hul.harvard.edu/10.1112/jlms/s1-26.3.179.

[Dav64] H. Davenport. "Corrigendum: "On a principle of Lipschitz"". In: *J. London Math. Soc.* 39 (1964), p. 580. ISSN: 0024-6107. DOI: 10.1112/jlms/s1-39.1.580-t. URL: https://doi-org.ezp-prod1.hul.harvard.edu/10.1112/jlms/s1-39.1.580-t.

[DH71] H. Davenport and H. Heilbronn. "On the density of discriminants of cubic fields. II". In: *Proc. Roy. Soc. London Ser. A* 322.1551 (1971), pp. 405–420. ISSN: 0962-8444. DOI: 10.1098/rspa.1971.0075. URL: https://doi-org.ezp-prod1.hul.harvard.edu/10.1098/rspa.1971.0075.

[Eke91] Torsten Ekedahl. "An infinite version of the Chinese remainder theorem". In: *Comment. Math. Univ. St. Paul.* 40.1 (1991), pp. 53–59. ISSN: 0010-258X.

[EV06] Jordan S. Ellenberg and Akshay Venkatesh. "The number of extensions of a number field with fixed degree and bounded discriminant". In: *Ann. of Math. (2)* 163.2 (2006), pp. 723–741. ISSN: 0003-486X. DOI: 10.4007/annals.2006.163.723. URL: https://doi.org/10.4007/annals.2006.163.723.

[Gra98] Andrew Granville. "*ABC* allows us to count squarefrees". In: *Internat. Math. Res. Notices* 19 (1998), pp. 991–1009. ISSN: 1073-7928. DOI: 10.1155/S1073792898000592. URL: https://doi-org.ezp-prod1.hul.harvard.edu/10.1155/S1073792898000592.

[Hoo67] C. Hooley. "On the power free values of polynomials". In: *Mathematika* 14 (1967), pp. 21–26. ISSN: 0025-5793. DOI: 10.1112/S002557930000797X. URL: https://doi-org.ezp-prod1.hul.harvard.edu/10.1112/S002557930000797X.

[Igu00] Jun-ichi Igusa. *An introduction to the theory of local zeta functions.* Vol. 14. AMS/IP Studies in Advanced Mathematics. American Mathematical Society, Providence, RI; International Press, Cambridge, MA, 2000, pp. xii+232. ISBN: 0-8218-2015-X.

[Ked07] Kiran S. Kedlaya. "Mass formulas for local Galois representations". In: *Int. Math. Res. Not. IMRN* 17 (2007). With an appendix by Daniel Gulotta, Art. ID rnm021, 26. ISSN: 1073-7928.

DOI: 10.1093/imrn/rnm021. URL: https://doi.org/10.1093/imrn/rnm021.

[Lev14] Friedrich Levi. "Kubische Zahlkörper und binäre kubische Formenklassen". In: *Berichte über die Verhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig, Mathematisch-Physische Klasse* 66 (1914), pp. 26–37.

[Mal02] Gunter Malle. "On the distribution of Galois groups". In: *J. Number Theory* 92.2 (2002), pp. 315–329. ISSN: 0022-314X. DOI: 10.1006/jnth.2001.2713. URL: https://doi.org/10.1006/jnth.2001.2713.

[Mal04] Gunter Malle. "On the distribution of Galois groups. II". In: *Experiment. Math.* 13.2 (2004), pp. 129–135. ISSN: 1058-6458. URL: http://projecteuclid.org/euclid.em/1090350928.

[ME05] M. Ram Murty and Jody Esmonde. *Problems in algebraic number theory*. Second. Vol. 190. Graduate Texts in Mathematics. Springer-Verlag, New York, 2005, pp. xvi+352. ISBN: 0-387-22182-4.

[Mer74] F. Mertens. "Ueber einige asymptotische Gesetze der Zahlentheorie". In: *J. Reine Angew. Math.* 77 (1874), pp. 289–338. ISSN: 0075-4102. DOI: 10.1515/crll.1874.77.289. URL: https://doi-org.ezp-prod1.hul.harvard.edu/10.1515/crll.1874.77.289.

[Sch68] Wolfgang M. Schmidt. "Asymptotic formulae for point lattices of bounded determinant and subspaces of bounded height". In: *Duke Math. J.* 35 (1968), pp. 327–339. ISSN: 0012-7094. URL: http://projecteuclid.org/euclid.dmj/1077377618.

[Sch95] Wolfgang M. Schmidt. "Number fields of given degree and bounded discriminant". In: 228. Columbia University Number Theory Seminar (New York, 1992). 1995, pp. 4, 189–195.

[Ser78] Jean-Pierre Serre. "Une "formule de masse" pour les extensions totalement ramifiées de degré donné d'un corps local". In: *C. R. Acad. Sci. Paris Sér. A-B* 286.22 (1978), A1031–A1036. ISSN: 0151-0509.

[Ser97] Jean-Pierre Serre. *Lectures on the Mordell-Weil theorem*. Third. Aspects of Mathematics. Translated from the French and edited by Martin Brown from notes by Michel Waldschmidt, With a foreword by Brown and Serre. Friedr. Vieweg & Sohn, Braunschweig, 1997, pp. x+218. ISBN: 3-528-28968-6. DOI: 10.1007/978-3-663-10632-6. URL: https://doi.org/10.1007/978-3-663-10632-6.

[Sie89] Carl Ludwig Siegel. *Lectures on the geometry of numbers*. Notes by B. Friedman, Rewritten by Komaravolu Chandrasekharan with the assistance of Rudolf Suter, With a preface by Chandrasekharan. Springer-Verlag, Berlin, 1989, pp. x+160. ISBN: 3-540-50629-2. DOI: 10.1007/978-3-662-08287-4. URL: https://doi-org.ezp-prod1.hul.harvard.edu/10.1007/978-3-662-08287-4.

[Wid10] Martin Widmer. "Counting primitive points of bounded height". In: *Trans. Amer. Math. Soc.* 362.9 (2010), pp. 4793–4829. ISSN: 0002-9947. DOI: 10.1090/S0002-9947-10-05173-1. URL: https://doi-org.ezp-prod1.hul.harvard.edu/10.1090/S0002-9947-10-05173-1.