Math 280Y: Arithmetic Statistics

Spring 2023

Syllabus

Course website: `https://fabiangundlach.org/23-spring/280Y/`

Instructor
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Prerequisites

This is a graduate topics course. Undergraduates are welcome.

Prerequisites are basic algebra and combinatorics (group actions, orbit–stabilizer theorem, ...), basic multivariable calculus, Galois theory, and algebraic number theory (number fields, local fields, geometry of numbers). We will use basic measure theory and Fourier analysis.

Some previous exposure to analytic number theory (in particular sieves, Dirichlet series) would be helpful.

Overview

In this course, we will count arithmetic objects (such as polynomials, number fields, or ideal classes). Several methods have been used in this area:

1. the geometry of numbers, orbit counting, sieves
2. the theory of Dirichlet series
3. in positive characteristic: the Grothendieck–Lefschetz fixed point formula

The main focus of the course will be on method 1. It has in recent years produced a number of interesting results and we try to give an overview and a unifying exposition of the important ideas. Method 2 will also be discussed. Method 3 will not be covered in any detail.

Many problems are still open, which we will approach heuristically.

Tentative list of topics

Random primes, random polynomials, counting local fields, counting number fields (Malle's conjecture), random number fields (Cohen-Lenstra heuristics), sieves, lattice-point and orbit-counting methods, heuristics. If time permits, we might also discuss random elliptic curves.

The focus will be on the use of geometry of numbers methods in arithmetic statistics.

References

There is no official textbook for this course, but here are some good first references:

- *Problems in Analytic Number Theory* by M. Ram Murty contains a nice introduction to Dirichlet series (chapters 1–3) and sieves (chapter 9).
- Melanie Matchett Wood's notes *Asymptotics for number fields and class groups* from the Arizona Winter School 2014: `http://swc.math.arizona.edu/aws/2014/index.html`

I'm not aware of a reference comprehensively treating the subjects of this course. There will be a list of additional references for specific topics on the course website.

Grading

There will be graded homework. Moreover, you will need to write a final paper (7-10 pages) on a topic related to the class material.

The final grade will be 60% based on homework and 40% on the final paper. The two lowest homework scores will be dropped.

You are encouraged to collaborate on homework, but have to write the solutions up independently. Please acknowledge collaborators and other sources.