Math 223b: Algebraic Number Theory

Spring 2021

Syllabus

Prerequisites

This is a graduate course on Algebraic Number Theory. Undergraduates are welcome.

The course is mostly independent from the previous course 223a, except for the theory of local fields.

Prerequisites are basic algebra (rings, modules, ...) and algebraic number theory (number fields, local fields; for example from courses 129 and 223a or the first two chapters of *Algebraic Number Theory* by JÜRGEN NEUKIRCH). We will also make use of some algebraic geometry (for example from course 137). If you know chapters I, II, IV of *Algebraic Geometry* by HARTSHORNE, that would be more than enough. Lastly, we will use a small amount of complex analysis.

Tentative list of topics

Elliptic curves, abelian varieties, Jacobian varieties, the Mordell-Weil theorem, Faltings's theorem, some results on diophantine approximation

Tablets

To discuss mathematics in sections, office hours, and among yourselves, please get a graphics tablet.

References

There is no official textbook for this course, but here are some good references:

- *Lectures on the Mordell–Weil theorem* by Jean-Pierre Serre
- *Introduction to abelian varieties* by V. Kumar Murty
- Lecture notes on *Abelian varieties* by James S. Milne: `https://www.jmilne.org/math/CourseNotes/av.html`
- *Siegel's theorem in the compact case* by Paul Vojta contains a simplified proof of Faltings's theorem
- *The Mordell conjecture revisited* by Enrico Bombieri contains an even more elementary proof of Faltings's theorem
- *Diophantine approximations and Diophantine equations* by Wolfgang M. Schmidt
- *Lecture notes on Diophantine analysis* by Umberto Zannier

I plan to put the lecture notes online after each class.

Grading

There will be weekly homework.

Furthermore, there will be a short final paper (7–10 pages). I will provide a list of possible topics, but you're also more than welcome to come up with your own ideas. There will be an opportunity to submit a draft before the final deadline for feedback.

The final grade will be 70% based on homework and 30% on the final paper. The two lowest homework scores will be dropped.

You are encouraged to collaborate on homework, but must write the solutions up independently. Remember to always acknowledge collaborators and other sources, both on homework assignments and the final paper.