Math 223a: Algebraic Number Theory

Fall 2020

Syllabus

Prerequisites

This is an introductory graduate class on Algebraic Number Theory. Undergraduates are welcome.

Prerequisites are basic algebra (rings, modules, ...) and topology (compactness, Hausdorffness, ...), Galois theory (fundamental theorem, normal/separable extensions, ...), and algebraic number theory (for example from course 129: number fields, ideal class groups, splitting behavior of primes). I recommend the first chapter of *Algebraic Number Theory* by Jürgen Neukirch (see below) if you need to catch up on number fields.

Tentative list of topics

Infinite Galois theory, valuations, local fields, higher ramification groups, local class field theory (the Lubin–Tate approach and Galois cohomology), overview of global class field theory (without proofs). If time allows, we might do one or two further topics, such as central simple algebras and Brauer groups.

Tablets

To discuss mathematics in sections, office hours, and among yourselves, please get a graphics tablet. You can contact `ipadrequest@fas.harvard.edu` to borrow an iPad with stylus from Harvard.

References

There is no official textbook for this course, but here are some good references:

- *Algebra—from the viewpoint of Galois theory* by Siegfried Bosch contains a nice introduction to infinite Galois extensions (chapter 4.2).
- *Algebraic Number Theory* by Jürgen Neukirch is a thorough introduction. It includes a lot of material on class field theory and many exercises. It doesn't do general Galois cohomology because Neukirch came up with an explicit and less abstract proof of class field theory. There are many clever exercises. If you know German, you can also read the original (*Algebraische Zahlentheorie*)!
- *Local fields* by Jean-Pierre Serre is a great, but somewhat terse, introduction to local fields.
- Bjorn Poonen wrote an excellent eight page summary of class field theory:
  `http://www-math.mit.edu/~poonen/papers/cft.pdf`
- James S. Milne wrote some detailed and very readable lecture notes on class field theory:
  `http://www.jmilne.org/math/CourseNotes/cft.html`
- *Class Field Theory – The Bonn Lectures* (German title: *Klassenkörpertheorie*) by Jürgen Neukirch treats the subject from the point of view of Galois cohomology.

Grading

There will be weekly homework.

Furthermore, there will be a short final paper (7–10 pages). I will provide a list of possible topics, but you're also more than welcome to come up with your own ideas. There will be an opportunity to submit a draft before the final deadline for feedback.

The final grade will be 70% based on homework and 30% on the final paper. The two lowest homework scores will be dropped.

You are encouraged to collaborate on homework, but must write the solutions up independently. Remember to always acknowledge collaborators and other sources, both on homework assignments and the final paper.