Math 223a: Algebraic Number Theory

Fall 2020

Problem set #11

due Friday, November 20 at 10:30am

Problem 1. Let $\pi \neq \pi'$ be two uniformizers of the same nonarchimedean local field K. Show that the corresponding Lubin–Tate modules F_π and $F_{\pi'}$ are not isomorphic as formal $\mathcal{O}_K$-modules.

Problem 2. Let $G = \{a, \sigma\}$ and define the G-module $\widetilde{\mathbb{Z}}$ as in class (where σ acts on the group $\widetilde{\mathbb{Z}} \cong \mathbb{Z}$ by multiplication by -1). Show that $H^1(G, \widetilde{\mathbb{Z}}) = \mathbb{Z}/2\mathbb{Z}$.

Problem 3. Let G be a finite cyclic group generated by σ. Show that the following sequence is exact, where $\varepsilon : \mathbb{Z}[G] \to \mathbb{Z}$ is the map sending $\sum_{g \in G} a_g g$ to $\sum_{g \in G} a_g$ and we denote the multiplication by $\sigma - 1$ map on $\mathbb{Z}[G]$ by $\sigma - 1$ and the multiplication by $N = \sum_{g \in G} g$ map on $\mathbb{Z}[G]$ by N:

$$0 \longleftarrow \mathbb{Z} \xleftarrow[\varepsilon]{} \mathbb{Z}[G] \xleftarrow[\sigma-1]{} \mathbb{Z}[G] \xleftarrow[N]{} \mathbb{Z}[G] \xleftarrow[\sigma-1]{} \mathbb{Z}[G] \xleftarrow[N]{} \cdots$$

Problem 4. For any finite field $\mathbb{F}_q$, let $\mathbb{F}_q^\times$ act on $\mathbb{F}_q$ by multiplication. Show that $H^1(\mathbb{F}_q^\times, \mathbb{F}_q) = 0$.