Math 223a: Algebraic Number Theory

Fall 2019

Syllabus

Location: Science Center 222 (FAS)

Course website:
`http://math.harvard.edu/~gundlach/19-fall/223a/`

Times: Tuesdays and Thursdays 12:00–1:15pm

Instructor	**Course assistant**
Fabian Gundlach	Kenz Kallal
`gundlach@math.harvard.edu`	`kenzkallal@college.harvard.edu`
Office hours:	*Office hours:*
Tu/Th 2–3pm in Science Center 233	TBD

Prerequisites

This is an introductory graduate class on Algebraic Number Theory. Undergraduates are welcome.

Prerequisites are basic algebra (rings, modules, ...) and topology (compactness, Hausdorffness, ...), Galois theory (fundamental theorem, normal/separable extensions, normal basis theorem, ...), and algebraic number theory (for example from course 129: number fields, ideal class groups, splitting behavior of primes, Chebotarev's density theorem would be useful). I recommend the first chapter of *Algebraic Number Theory* by Jürgen Neukirch (see below) if you need to catch up on number fields. The Chebotarev density theorem is Theorem VII.13.4. (You don't need to read everything before to understand at least the statement.)

Tentative list of topics

Infinite Galois theory, valuations, local fields, higher ramification groups, local class field theory (the Lubin–Tate approach and Galois cohomology), overview of global class field theory (without proofs). If time allows, we might do one or two further topics.

References

There is no official textbook for this course, but here are some good references:

- Bjorn Poonen wrote an excellent eight page summary of class field theory:
  http://www-math.mit.edu/~poonen/papers/cft.pdf
- *Local fields* by Jean-Pierre Serre is a great, but somewhat terse, introduction to local fields.
- James S. Milne wrote some detailed and very readable lecture notes on class field theory:
  http://www.jmilne.org/math/CourseNotes/cft.html
- *Algebraic Number Theory* by Jürgen Neukirch is a thorough introduction. It includes a lot of material on class field theory and many exercises. It doesn't do general Galois cohomology because Neukirch came up with an explicit and less abstract proof of class field theory. There are many clever exercises. If you know German, you can also read the original (*Algebraische Zahlentheorie*)!
- *Class Field Theory – The Bonn Lectures* (German title: *Klassenkörpertheorie*) by Jürgen Neukirch treats the subject from the point of view of Galois cohomology.
- *Algebra—from the viewpoint of Galois theory* by Siegfried Bosch contains a nice introduction to infinite Galois extensions (chapter 4.2).

Grading

There will be weekly homework, usually due Thursdays at noon. You can bring it to class or send it to the course assistant by email:
kenzkallal@college.harvard.edu

Furthermore, there will be a short final paper (5–10 pages) on a topic related to the class material.

The final grade will be 70% based on homework and 30% on the final paper. The two lowest homework scores will be dropped.

You are encouraged to collaborate on homework, but write the solutions up independently. Please acknowledge collaborators and other sources.